Froot and Stein explain convex costs of external financing following Froot et. al., i.e. with a variant of the costly state verification models of Townsend and Gale and Hellwig.1

This brief note suggests that the underinvestment problem identified by Myers and Majluf may be considered as an alternative cause for convex costs of external financing and hence risk aversion in earlier periods.2 The text introduces a slightly amended version of the Myers and Majluf base case and describes an example that supports the convex costs of external financing hypothesis.

The Myers and Majluf based motivation for convex costs of external funds relies on effects of equity financing. Hence costs of external financing can be convex although debt is assumed to be risk-free. The following describes how such debt could be constructed although it finances levered investments into assets with normally distributed returns.

Assumed return distribution and the company’s financing sources

In T-1, a company has available equity amounting to $K$. The company can raise additional equity, which will be discussed further below, and also borrow an amount up to a certain multiple of its equity, in the form of zero coupon bonds earning the risk free rate. This assumption can be justified if the value of the company’s portfolio of assets can frequently and without costs be monitored by the creditors, and the debt contract is constructed such that as soon as the portfolio market value has dropped by an amount equal to the value of the company’s equity, the creditors take over the portfolio.3

In every instant, an asset’s market value moves either a specific amount up or a specific amount down, i.e. the instantaneous return has a binomial distribution. This implies that returns over a (short) period

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2 While Froot and Stein don’t directly draw on Myers and Majluf as justification for risk aversion, they highlight that the two models share the implication that holding financial slack may avoid underinvestment, see Froot and Stein, p. 62.

3 Even if the portfolio is completely illiquid, as assumed in the next section, it is here considered to have a measurable value, determined by an asset pricing model as also described in the next section.
(here the period between $T-1$ and $T$) that is divided into a large number of instants approach a normal distribution when the number of instants per short period approaches infinity.\textsuperscript{4}

Assume the creditors can monitor the company’s portfolio market value (and react if needed) at several points in time between $T-1$ and $T$. Between each of these monitoring points lie short time intervals, consisting of a certain number of instants. With the potential up and down moves of every asset in an instant being known, the maximal borrowing amount can be determined such that the available equity equals the maximum possible portfolio loss between two monitoring points. Then, and with creditors being able to react after every instant and take over the asset portfolio if equity is wiped out, their potential loss approaches zero, and the debt becomes eventually riskless, although the asset value may be a multiple of the equity, and asset returns over short time intervals are described by a (multivariate) normal distribution.\textsuperscript{5} Assume for the following that the maximum possible portfolio loss between two monitoring points is 20%, implying a maximum for the asset value to equity ratio of 500%.

\textbf{Initial portfolio}

The company holds a position in the riskless asset; the size of the position is equal to the company’s capital $K$ and the return of the position until $T$ is the risk-free rate of interest $r$.\textsuperscript{6}

The company further holds a portfolio of (perfectly) illiquid\textsuperscript{7}, financed (legacy) exposures to risky assets with aggregate exposure size $O$.

The cash flow the company will receive in $T$ from these investments (after deducting the financing costs) is $CF_O$:

$$CF_O = K(1 + r_p) = K[1 + r + w_O(r_O - r)]$$

with

$$w_O = \frac{O}{K}$$

There will be no further cash flows from the illiquid assets, i.e. after the cash flow in $T$ is paid, the assets are worthless (or cease to exist). The size of the existing portfolio is assumed to be larger than $K$ but smaller than the maximum possible exposure of $K$ times five. Hence, the company has capital available


\textsuperscript{5} Alternatively one could start the reasoning with assuming that the instantaneous return is at least approximately normally distributed, and argue that the (infinitesimal small) instant can be divided into a number of (“even smaller”) infinitesimal small “sub-instants” with a binomial distributed return, and the number of sub instants per instant approaches infinity. (While the instant contains a large number of sub instants, these are so small, that the instant itself is an infinitesimal small period.) This may however be inconsistent with a definition of an instant as being the absolute smallest period, against which one might want to argue, that infinitesimal small rules out that an “absolute” minimum size period exists. These questions are however beyond the scope of this text.

\textsuperscript{6} For consistency with the normal distribution assumption, the period between $T-1$ and $T$ is assumed to be very short.

\textsuperscript{7} (Perfectly) illiquid means here, that these exposures are not tradeable at all.
to support further borrowing and investment into risky assets. Following Myers and Majluf the maximum amount of “financial slack” the company can attain with its unused capital is in the following labelled $S$ and can be calculated as:\(^8\)

$$S = S \left( K - \frac{O}{S} \right) = 5K - O$$

**Market value of the company’s equity**

The company’s shares are traded on a market with the following pricing model:

$$V = \frac{E(CF) - \lambda \text{cov}(r_M, CF)}{1 + r}$$

Where $CF$ is a cash flow in $t=T$, $V$ is the $T-1$ market value of the cash flow, $M$ is the return of a portfolio representing a priced factor\(^9\), $r$ is the discount rate for certain $t=T$-cash flows and $\lambda$ is the market price of risk, defined as follows:

$$\lambda = \frac{\mu_M - r}{\sigma_M^2}$$

where $\mu_M$ is expected value of the return $r_M$ corresponding to the priced factor, and $\sigma_M$ is the volatility of that return.

As $CF_0$ is the cash flow to the company after deducting the financing costs, it is the cash flow to shareholders and its market value $K + a$ also equals the market value of the firm’s equity – calculated with above pricing model. With

$$a = O + \frac{O (\mu_D - r - \lambda \text{cov}(r_O, r_M))}{1 + r}$$

being the present value of the cash flows from the financed portfolio of risky legacy exposures plus $\frac{O}{S}$, the present value of the equity allocated to secure the loan that finances the portfolio, the market value of the company’s equity stake in its initial portfolio can be written as:

$$\frac{K(1 + \mu_p) - K\lambda \text{cov}(r_p, r_M)}{1 + r} = K + O (\mu_D - r - \lambda \text{cov}(r_O, r_M)) = \frac{S + O (\mu_D - r - \lambda \text{cov}(r_O, r_M))}{S + O} \frac{S}{5} + a$$

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\(^8\) One difference to the Myers and Majluf base case here is though, that in the latter there is only equity financing – and $S$ hence represents unused equity, while here it represents an amount of debt financing that can be secured with the unused portion of equity $K$.

\(^9\) Factor is here to be understood strictly on an ex ante basis, i.e. a factor determining expected returns only. There is no assumption about factors driving realized returns. Applying the combined approach in the post on the CCAPM with heterogeneous expected returns and illiquid assets in this blog, the tradeable factor would be the return of the aggregate of investor’s investment portfolios with maximum correlation with their optimal consumption at the end of the instant.
New investment in T-I

The only investment opportunity available with a positive expected excess return after adjusting for priced risk is a fixed-size, large, illiquid investment, for example the takeover of another firm. The T-I net present value of this new investment is in the following labelled $b$:

$$b = I \left( E(r_b - r) - \lambda \text{cov}(r_M, r_b) \right) \frac{1}{1 + r}$$

where $I$ is the invested amount. The company may or may not have enough equity available at T-I to attain sufficient financing for the investment, and may or may not decide to issue additional equity. If the company does not issue additional equity and hence will not invest into the large illiquid investment, it will invest any available funds into the riskless asset – which is here, with both borrowing and lending being done at $r$, equivalent to partially repaying the loan that finances the initial portfolio.

The Myers and Majluf underinvestment problem

While management knows the true net present values of the existing and new investments at T-I, current and potential future shareholders, i.e. “the market” doesn’t. The management acts in the interest of current shareholders. When deciding about issuing equity, it therefore compares the value of the current owners’ share of the true (intrinsic) value of the assets with the value of the current owners’ share of the intrinsic value when additional equity is issued. The share of the company held by current owners after an equity issue depends on the market price of the total number of existing shares of the company at the time of issue, and the amount of new equity issued. If no new shares are issued, the value of existing shares is:

$$V_{old}^{T-1} = \frac{S}{S} + a$$

If management decides to issue and invest into the new asset, the intrinsic value of the old shares is:

$$V_{old}^{T-1} = \frac{P}{P+E} V_{T-1} = \frac{P}{P+E} (E + K + a + b)$$

Where $V_{T-1}$ is the total intrinsic value, and:

$$P+E = A(M') + B(M') + K + E$$

Or:

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10 Adjusting for priced risk here means subtracting the fair expected return for the asset according to the pricing model from the asset’s expected return.

\[ P' = A(M') + B(M') + K \]

Where \( A(M') \) and \( B(M') \) are the values expected by the market for \( a \) and \( b \) respectively, conditional on new shares being issued.\(^{12}\) Assuming old shareholders are passive, i.e. if they will not participate in the issue, the condition to issue is:\(^{13}\)

\[
\frac{S}{5} + a \leq \frac{P'}{P' + E} (E + K + a + b)
\]

However, when the management announces that it intends to issue new equity, the market knows that this condition is true, market participants will adjust their expectations, i.e. \( A \) and \( B \) are updated, and the share price \( P' \) for old and new shares adjusts immediately. Hence, management will also check if the decision to issue would still be in the interest of existing shareholders after \( P' \) has adjusted. This is in turn known by shareholders, and will also be incorporated in the adjustment of their expectations. Myers and Majluf describe this process by way of example, where management first checks the condition based on the current share price, then adjusts \( A \) and \( B \) to account for shareholders being able to rule out values for \( a \) and \( b \) that would lead to the decision not to issue, checks the condition again – and repeats the process until \( P' \) converges.\(^{14}\)

As can be seen from the condition to issue, maximization of the value of the old shareholders’ position implies that if new equity has to be issued to finance a positive NPV investment, the investment will not necessarily be undertaken, leading to a loss in the form of a missed opportunity to increase the intrinsic value, that would be undertaken if sufficient internal funds had been available. Management’s described anticipation of market reactions to their decision amplifies this underinvestment as shown by Myers and Majluf with discrete examples and numerical analysis,\(^{15}\) i.e. underinvestment will happen in additional states of the world due to this anticipation. Myers and Majluf refer to the expected value of loss in the form of missed opportunities to invest in positive NPV assets (or projects) in \( T-2 \), i.e. when management knows \( A \) and \( B \), but not \( a \) and \( b \) yet, as “ex ante loss”\(^{16}\).

**Ex ante loss as a convex function of internal funds**

Numerical analysis shows that the ex ante loss may be a convex function of the amount \( E \) of equity raised, at least for some values and parameters of the distributions of \( a \) and \( b \). For example the first chart below shows the ex ante loss \( L \) as a function of issue size \( E \) for a given investment amount \( I = 250 \),

\(^{12}\) One period earlier, at \( T-2 \), management also knows only \( A \) and \( B \) – this will be relevant further below.

\(^{13}\) The assumption that old shareholders are passive will be discussed in a revised version of this post.

\(^{14}\) Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have, Stewart C. Myers, Nicholas S. Majluf, Journal of Financial Economics 13 (1984) 187-221, p. 205. On the same page Myers and Majluf state that there is no guarantee for a unique (stable) \( P' \), but that there will be at least one. Hence, strictly speaking, for a comprehensive analysis one would probably have to start the iterative approach with a variety of values for \( P' \), not just the initial one.

\(^{15}\) Myers and Majluf, p. 192-194 and 205-207

\(^{16}\) Myers and Majluf, p. 215
with $a$ and $b$ being lognormally distributed with standard deviation of their respective natural logarithms being 40 and 30 percent respectively, correlation of the normal variables being 0.3, expected values $A$ and $B$ of approx. 100 and 20 respectively, and unused equity $K-O/5$ ranging from 0 to 50.\(^\text{17}\)

As the chart indicates\(^\text{18}\), the ex ante loss is (at least approximately) a convex function of the issue size.

From the perspective of the company’s management, the $T-2$ PV of the company, i.e. the discounted value of $V = A + B + K - L$, is hence a concave function of internal funds or equity $K-O/5$ available in $T-1$, as the following chart visualizes, which is based on the same simulation as the chart above:\(^\text{19}\)

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\(^{17}\) Links to download the spreadsheet and VBA macro used to perform the analysis are given in the blog post that contained also the link to this note.

\(^{18}\) And as can be seen in more detail from the differences between values of $L$ for varying $K-O/5$ as shown in the spreadsheet, A9:A16.

\(^{19}\) Strictly speaking, the function becomes linear for values of $S$ that exceed the required investment. In the original setting of Froot and Stein and Froot et al. the new investment has a concave payoff function, so that concavity is ensured also for very large values of internal funds. With the assumption here of the new investment having a fixed size, an alternative explanation for concavity over the whole range would have to be found. However, it may be sufficient to assume concavity over a wide range to explain risk aversion – this will be discussed in more detail in a revised version of this post. Alternatively, one may just rule out very large values of internal funds as an approximating solution.
The amount of unused equity or internal funds in $T-1$ is a function of the portfolio return between $T-2$ and $T-1$. In other words, the target function the bank maximizes in $T-2$ is a concave function of internal wealth available in $T-1$, and the bank will correspondingly hedge all tradeable and fairly priced exposures in $T-2$, and mean-variance optimize their portfolio of untradeable and/or unfairly priced exposures like a risk-averse investor.

**Limitations**

Above results are for a specific set of parameter values, i.e. volatilities, correlations and (net) present values. As can be seen with the accompanying spreadsheet, other parameter values lead to a different shape of the ex ante loss as a function of the issue size; and the ex ante loss function may be linear or even concave over certain ranges. Nevertheless, while further analysis appears necessary to understand these properties in more detail, the above example shows, that the Myers and Majluf problem may at least in some cases lead to corporate risk aversion.

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20 Note further, that negative $K-O/5$ is ruled out in Myers and Majluf – p.198. This can be justified with a similar set of assumptions and reasoning as in the initial section. Summarizing this and the previous footnote, and with default being ruled out here as explained in the initial section, the company value is at least approximately concave over the range of attainable values of $S$, which is a function of a normally distributed portfolio return between $T-2$ and $T-1$.

21 For details on hedging and investment policy see the previous two posts in this blog. Note that there is a special case that has not been discussed, yet in this blog, which is an active view on the priced factor – this case will be reviewed in a later post. To be consistent with Froot and Stein, the illiquid assets discussed earlier are also assumed to be part of the portfolio at $T-2$, and with a normally distributed market value at $T-1$ which is in part paid as a (normally distributed) cash flow.